

Supplementary material to the work: “Brownian Bridge for Coherent State Path Integral Monte Carlo”

Riccardo Fantoni*

Università di Trieste, Dipartimento di Fisica, strada Costiera 11, 34151 Grignano (Trieste), Italy

(Dated: April 27, 2026)

Supplementary material to the article “Brownian Bridge for Coherent State Path Integral Monte Carlo”.

Keywords: Quantum Many Body; Coherent States; Path Integral; Quantum Monte Carlo; Brownian Bridge; Helium; Thermodynamics

I. INTRODUCTION

We give in the Appendix the code listing for the CSPIMC simulations carried out in the work: “Brownian Bridge for Coherent State Path Integral Monte Carlo”. We will denote this paper with [*].

Subroutine `cs()` calculates the high temperature reduced density matrix of Eq. (3.7) of paper [*]. Subroutine `bridge()` performs the Brownian bridge of Eqs. (4.1)-(4.2) of the paper [*] and subroutine `accs()` calculates the “a priori” sampling distribution of Eq. (4.6) of the paper [*].

Appendix A: The code

This is the code used for the CSPIMC computer experiment. We list here the main FORTRAN code `cspimc.f` with its included `mc-cs.par` parameters file. And the input data file `data-cs-new.in` that is read at the beginning of the run.

```
-----
cspimc.f
-----
PROGRAM CoStPIMC
IMPLICIT NONE
C *****
C ** MONTE CARLO SIMULATION PATH INTEGRAL WITH COHERENT STATES **
C **
C ** space dimensions: DIM=1,2,3 **
C ** number of particles: NP **
C ** number of timesteps: FTNO **
C ** units: hbar=k_B=1 **
C ** BETA=1/TEMP=FTNO*LS (LS imaginary time spacing) **
C ** Harmonic Oscillator (HO) for Coherent States (CS): **
C ** MOHO = m_ho*omega **
C **
C ** OBSERVABLES: **
C ** kinetic energy KE **
C ** potential energy V **
C *****
INCLUDE 'mc-cs.par' ! parameters

INTEGER*4 SEED
INTEGER*8 IDIM, STEP, I, J, K
INTEGER*8 II(MNP), JJ, KK, PREV(MNP)
INTEGER*8 C1, CP, IP, KP, NIP, NKP, LL, OUT1, OUT2, PIP
INTEGER*8 INIT, NSTEP, IPRINT, ISAVE, IRATIO, IEQUI
INTEGER*8 MBMM, MCM

REAL*8 DENS, TEMP, BETA, DENSLJ
REAL*8 DRMAX, ALPHA, CDIM
REAL*8 RANF, DUMMY, SR9, SR3
REAL*8 VLRC, VLRC6, VLRC12, WLRC, WLRC6, WLRC12
REAL*8 ACM, ACATMA, ACATMAS, ACATMAB
REAL*8 V, VNEW, WOLD, VEND, DELTV, VN, VS
REAL*8 W, WNEW, WOLD, WEND, DELTW, WN, WS
REAL*8 KE, KENEW, KEOLD, DELTKE, KEEND
REAL*8 AVV, ACV, ACVSQ, FLV
REAL*8 AVKE, ACKE, ACKESQ, FLKE
```

```
REAL*8 ACT, ACTNEW, ACTOLD, DELTACT, DELTACTB
REAL*8 RXIOLD(MDIM), RXINEW(MDIM)
REAL*8 RAOLD(MDIM), PAOLD(MDIM)
REAL*8 RBOLD(MDIM), PBOLD(MDIM)
REAL*8 RANEW(MDIM), PANEW(MDIM)
REAL*8 RBNEW(MDIM), PBNEW(MDIM)
REAL*8 RXP(MDIM,0:N), RXPP(MDIM,0:N)
REAL*8 RAP(MDIM,0:N), RAPP(MDIM,0:N)
REAL*8 RBP(MDIM,0:N), RBPP(MDIM,0:N)
REAL*8 STD, PS, KKK, VVV, WWW
REAL*8 RATIO, RATIOS, RATIOB

COMPLEX*16 ZNEW, ZOLD, ZN, ZO

CHARACTER CNFILE*30, POTK*10

LOGICAL OVRLAP, IFB, IFDISP, IFBRIDGE, IFZERO

C PI = ACOS(-1.d0)
C *****
C ** READ INPUT DATA **
C *****
WRITE(*,(' ***** PROGRAM PIMC ***** '))
WRITE(*,(' COHERENT STATES '))
WRITE(*,(' PATH INTEGRAL MONTE CARLO PROGRAM '))
OPEN(UNIT=10, FILE='data-cs-new.in', STATUS='UNKNOWN')
READ(10,*) I
READ(10,*) DIM
READ(10,*) I
READ(10,*) SEED
READ(10,*) I
READ(10,*) IFB
READ(10,*) I
READ(10,*) NP
READ(10,*) I
READ(10,*) POTK
READ(10,*) I
READ(10,*) FTNO
READ(10,*) I
READ(10,*) FTM
READ(10,*) I
READ(10,*) NSTEP
READ(10,*) I
READ(10,*) IPRINT
READ(10,*) I
READ(10,*) ISAVE
READ(10,*) I
READ(10,*) IEQUI
READ(10,*) I
READ(10,*) IRATIO
READ(10,*) I
READ(10,*) CNFILE
READ(10,*) I
READ(10,*) INIT
READ(10,*) I
READ(10,*) DENS
READ(10,*) I
READ(10,*) TEMP
READ(10,*) I
READ(10,*) ALPHA
READ(10,*) I
READ(10,*) MBMM
READ(10,*) I
READ(10,*) RCUT
READ(10,*) I
READ(10,*) SIG
READ(10,*) I
READ(10,*) EPSR
READ(10,*) I
READ(10,*) EPSA
READ(10,*) I
READ(10,*) EPS
READ(10,*) I
READ(10,*) MOHO
READ(10,*) I
READ(10,*) IFDISP
READ(10,*) I
READ(10,*) IFBRIDGE
READ(10,*) I
READ(10,*) IFZERO
CLOSE(UNIT=10)
WRITE(*,(' IN DIMENSION (1,2,3,...)''I12/')) DIM
WRITE(*,(' ***** '))
GOTO 1111 ! comment if input from keyboard
```

* riccardo.fantoni@scuola.istruzione.it

```

WRITE(*,(' ***** PROGRAM PIMC ***** '))
WRITE(*,(' COHERENT STATES '))
WRITE(*,(' PATH INTEGRAL MONTE CARLO PROGRAM '))

WRITE(*,(' ENTER THE SPATIAL DIMENSIONS '))
READ(*,*) DIM
WRITE(*,(' ENTER SEED FOR RANDOM SEQUENCE '))
READ(*,*) SEED
WRITE(*,(' IF BOSE (T/F) '))
READ(*,*) IFB
WRITE(*,(' ENTER THE NUMBER OF PARTICLES < NMP '))
READ(*,*) NP
WRITE(*,(' ENTER TYPE OF PAIR POTENTIAL (Many Body) '))
READ(*,*) POTK
WRITE(*,(' ENTER THE NUMBER OF DISCRETIZATIONS < N '))
READ(*,*) FTNO
WRITE(*,(' ENTER THE BARE MASS '))
READ(*,*) FTM
WRITE(*,(' ENTER NUMBER OF CYCLES '))
READ(*,*) NSTEP
WRITE(*,(' ENTER NUMBER OF STEPS BETWEEN OUTPUT LINES '))
READ(*,*) IPRINT
WRITE(*,(' ENTER NUMBER OF STEPS BETWEEN DATA SAVES '))
READ(*,*) ISAVE
WRITE(*,(' ENTER NUMBER OF STEPS FOR EQUILIBRATION '))
READ(*,*) IEQUI
WRITE(*,(' ENTER INTERVAL FOR UPDATE OF MAX. DISPL. '))
READ(*,*) IRATIO
WRITE(*,(' ENTER THE CONFIGURATION FILE NAME '))
READ(*,*) CNFILE
WRITE(*,(' ENTER 0 IF INITIALIZATION NEEDED '))
READ(*,*) INIT
WRITE(*,(' ENTER THE DENSITY '))
READ(*,*) DENS
WRITE(*,(' ENTER THE TEMPERATURE '))
READ(*,*) TEMP
WRITE(*,(' ENTER MAX. DISPLACEMENT DRMAX/SIGMA '))
READ(*,*) ALPHA
WRITE(*,(' ENTER MAXIMUM NUMBER OF BRIDGE TIMESLICES '))
READ(*,*) MBMM
WRITE(*,(' ENTER THE POTENTIAL CUTOFF DISTANCE ( ^ 2.5) '))
READ(*,*) RCUT
WRITE(*,(' ENTER SIG '))
READ(*,*) SIG
WRITE(*,(' ENTER EPSR '))
READ(*,*) EPSR
WRITE(*,(' ENTER EPSA '))
READ(*,*) EPSA
WRITE(*,(' ENTER EPS '))
READ(*,*) EPS
WRITE(*,(' ENTER HO MASS*OMEGA=SQRT(MASS*k) '))
READ(*,*) MOHO
WRITE(*,(' IF DISPLACEMENT MOVE (T/F) '))
READ(*,*) IFDISP
WRITE(*,(' IF BRIDGE MOVE (T/F) '))
READ(*,*) IFBRIDGE
WRITE(*,(' IF ZERO PATH INITIALLY (T/F) '))
READ(*,*) IFZERO
WRITE(*,(' IN DIMENSION (1,2,3,...) ')) DIM
WRITE(*,(' ***** '))

1111 CONTINUE

IF (MBMM.GT. FTNO) THEN
  WRITE(*,*) "number of timeslices in bridge > ",FTNO
  STOP
ENDIF
RX=0.          ! real paths on the origin
RA=0.          ! ghost paths
RB=0.          ! ghost paths
PA=0.          ! ghost paths
PB=0.          ! ghost paths

C ** INITIALIZE RANDOM NUMBER GENERATOR **

CALL SRAND ( SEED )
IF (SEED.EQ.1) THEN
  CALL SRAND ( 0 )
ENDIF

C ** INVERSE TEMPERATURE BETA **

BETA = (1.d0/TEMP)

C ** IMAGINARY TIMESTEP **

LS = BETA/FTNO      ! time step
STD = (LS/FTM)*0.5 ! standard deviation of free-particle rho

C ** WRITE INPUT DATA **

WRITE(*,(' SEED ')) SEED
WRITE(*,(' POTENTIAL ')) POTK
WRITE(*,(' DIMENSIONS ')) DIM
WRITE(*,(' NUMBER OF PARTICLES ')) NP
WRITE(*,(' NUMBER OF CYCLES ')) NSTEP
WRITE(*,(' NUMBER OF EQUIL. STEPS ')) IEQUI
WRITE(*,(' OUTPUT FREQUENCY ')) IPRINT
WRITE(*,(' SAVE FREQUENCY ')) ISAVE
WRITE(*,(' RATIO UPDATE FREQUENCY ')) IRATIO
WRITE(*,(' CONFIGURATION FILE NAME ')) CNFILE
WRITE(*,(' TEMPERATURE ')) TEMP
WRITE(*,(' DENSITY ')) DENS
WRITE(*,(' PARTICLE MASS ')) FTM
WRITE(*,(' HO MASS*OMEGA ')) MOHO
WRITE(*,(' # TIME SLICES ')) FTNO
WRITE(*,(' TIME STEP ')) LS

C ** CONVERT INPUT DATA TO PROGRAM UNITS **

SIGMA = ( DBLE ( NP ) / DENS ) ** ( 1.0 / DIM )

DRMAX = ALPHA * SIGMA

RATIO = 0.0
RATIOS = 0.0
OUT1 = 0
OUT2 = 0

IF (POTK .EQ. 'LJ') THEN
  DENSLJ = DENS * SIG ** DBLE( DIM )
  RCUT = SIGMA/2.d0/SIG
ENDIF

C ** INITIALIZE CONFIGURATION **

DO I = 1, NP
  NEXT(I) = I
  LEXT(I) = I
  DO J = 1, FTNO
    MEXT(J,I) = I
  ENDDO
ENDDO

IF ( INIT .EQ. 0 ) THEN
  PRINT*, "PATHS INITIALIZATION ....."
  CALL INITCN ( CNFILE )      ! random configuration
  CALL READCN ( CNFILE )     ! random configuration

  IF (IFZERO) THEN
    RX=0.          ! real paths on the origin
    RA=0.          ! ghost paths
    RB=0.          ! ghost paths
    PA=0.          ! ghost paths
    PB=0.          ! ghost paths
  ENDF

C ** READ INITIAL CONFIGURATION **

IF ( INIT .NE. 0 ) THEN
  CALL READCN ( CNFILE )
ENDIF

C ** ZERO ACCUMULATORS **

ACV = 0.0
ACVSQ = 0.0
FLV = 0.0
ACKE = 0.0
ACKESQ = 0.0
FLKE = 0.0
ACM = 0.0
ACATMA = 0.0
ACATMAB = 0.0
ACATMAS = 0.0

C ** CALCULATE LONG RANGE CORRECTIONS **
C ** SPECIFIC TO THE LENNARD JONES FLUID **

IF (DIM .EQ. 1) CDIM = 1
IF (DIM .EQ. 2) CDIM = PI
IF (DIM .EQ. 3) CDIM = 2*PI

SR3 = - RCUT ** ( - 6. + DIM ) / ( - 6. + DIM )
SR9 = - RCUT ** ( - 12. + DIM ) / ( - 12. + DIM )

VLCR12 = 4 * EPS * CDIM * DENSLJ * NP * SR9
VLCR6 = - 4 * EPS * CDIM * DENSLJ * NP * SR3
VLCR = VLCR12 + VLCR6
WLCR12 = 4.0 * VLCR12
WLCR6 = 2.0 * VLCR6
WLCR = WLCR12 + WLCR6

C ** WRITE OUT SOME USEFUL INFORMATION **

WRITE(*,(' SIGMA ')) SIGMA
WRITE(*,(' MAXIMUM DISPLACEMENT ')) DRMAX

C ** CALCULATE INITIAL ENERGY AND CHECK FOR OVERLAPS **

CALL CSSUMUP (POTK, OVRLAP, KE, V)

IF (POTK .EQ. 'LJ') THEN
  VS = ( V + WLCR )
  WS = ( W + WLCR )
ELSE
  VS = V
ENDIF

WRITE(*,(' INITIAL V ')) VS
WRITE(*,(' INITIAL KE ')) KE

WRITE(*,(' START OF MARKOV CHAIN '))
WRITE(*,(' MOVE RATIO ACTION '))

C *****
C ** LOOPS OVER ALL CYCLES AND ALL TIME SLICES **
C *****

DO 100 STEP = 1, NSTEP

  CP = INT(FTNO*РАНF(DUMMY))+1 ! select a random timeslice
  MBM = INT((MBMM-1)*РАНF(DUMMY))+2 ! # timeslices in bridge
  IP = INT(NP*РАНF(DUMMY))+1 ! select a particle
  KP = INT(NP*РАНF(DUMMY))+1 ! select another particle
  IF (.NOT.IFB) KP = IP ! for boltzmann statistics
  IF (IFDISP) GOTO 77

C *****
C ** BRIDGE&SWAP MOVE (BOSE STATISTICS) **
C *****
  mcm = cp+mbm-floor((cp+mbm-.1)/ftn0)*ftn0

```



```

else
do c1=cp+1,ftn0
  ll=ll+1
  call cs(rapp(:,ll),pa(:,c1,kp),rbpp(:,ll)
:      ,pb(:,c1,kp),rxpp(:,ll),rx(:,c1+1,kp),zo)
:      znew=znew*zo
:      call cs(rapp(:,ll),pa(:,c1-1,kp),rbpp(:,ll)
:      ,pb(:,c1-1,kp),rx(:,c1-1,kp),rxpp(:,ll),zo)
:      znew=znew*zo
enddo
do c1=1,mcm-1
  ll=ll+1
:      call cs(rapp(:,ll),pa(:,c1,nip),rbpp(:,ll)
:      ,pb(:,c1,nip),rxpp(:,ll),rx(:,c1+1,nip),zo)
:      znew=znew*zo
:      call cs(rapp(:,ll),pa(:,c1-1,nip),rbpp(:,ll)
:      ,pb(:,c1-1,nip),rx(:,c1-1,nip),rxpp(:,ll),zo)
:      znew=znew*zo
enddo
endif
endif

call acccs(ps,ip,kp,cp,rxp,rxpp,rap,rapp,rbp,rbpp)
ps=ps*exp(-ls*(vnew-vold))*dble(znew)/dble(zold)

if (ps.gt.ranf(dummy)) then
  acatmab = acatmab + 1.d0
  call update(cp,rxp,rap,rbp,ip,nkp)
  if (ip.ne.kp) then
    call update(cp,rxpp,rapp,rbpp,kp,nip)
    call swap(cp,ip,nip,kp,nkp)
    call switch(next(ip),next(kp))
    call switch(next(cp,ip),next(cp,kp))
    acatmas = acatmas + 1.d0
  endif
endif

keneu=0.d0
do k=1,np
  if(k.eq.ip.or.k.eq.kp.or.k.eq.nip.or.k.eq.nkp)then
    do i=1,ftn0
      call cskkenergy ( k, i, kkk )
      keneu=keneu+kkk
    enddo
  endif
enddo

ke=ke+keneu-keold
vsv=vnew-vold
wsw=wnew-wold
endif

c build the permutations cycles in lext
call permcy(c)

7777 continue

IF (STEP.GT.IEQUI) THEN
  ACM = ACM + 1.0

C ** CALCULATE INSTANTANEOUS VALUES **

  IF (POTK .EQ. 'LJ') THEN
    VN = ( V + VLRC )
  ELSE
    VN = V
  ENDF

C ** ACCUMULATE AVERAGES **

  ACV = ACV + VN
  ACVSQ = ACVSQ + VN*VN
  ACKE = ACKE + KE
  ACKESQ = ACKESQ + KE*KE
  ENDF

  IF (IFBRIDGE) GOTO 97

C *****
C ** ENDS BRIDGE&SWAP MOVE **
C *****

77 CONTINUE

C *****
C ** DISPLACEMENT MOVE **
C *****

C ** PREVIOUS IP **

  DO I=1,NP
    J=NEXT(I)
    PREV(J)=I
  ENDDO

  NIP = NEXT(IP)
  PIP = PREV(IP)

  DO 90 C1 = 1, FTNO ! exact needs previous IP
C   DO 90 C1 = 2, FTNO ! exact
C   DO 90 C1 = 2, FTNO-1 ! exact

  DO IDIM = 1, DIM
    RXIOLD(IDIM) = RX(IDIM,C1,IP)
    raold(idim) = ra(idim,c1,ip)
    rbold(idim) = rb(idim,c1,ip)
    paold(idim) = pa(idim,c1,ip)
    pbold(idim) = pb(idim,c1,ip)
  ENDDO

C ** CALCULATE THE ENERGY OF I IN THE OLD CONFIGURATION **

```

```

CALL PENERGY ( POTK, RXIOLD, IP, C1,
VOLD, WOLD )

```

```

CALL CSKENERGY ( PAOLD, PBOLD, IP, C1,
KEOLD )

```

```

call cs(raold,paold,rbold,pbold,
rxibold,rx(:,c1+1,ip),zold)
call cs(raold,paold,rbold,pbold,
rx(:,c1-1,ip),rxibold,zo)
zold=zold*zo

```

```

C ** INSTANTANEOUS VALUE OF THE ACTION **
ACTOLD = KEOLD + VOLD

```

```

C ** MOVE I AND PICKUP THE CENTRAL IMAGE **

```

```

DO IDIM = 1, DIM
  RXINEW(IDIM) = RXIOLD(IDIM) +
( 2.0 * RANF ( DUMMY ) - 1.0 )*DRMAX
  RXINEW(IDIM) = RXINEW(IDIM) +
  DNINT ( RXINEW(IDIM)/SIGMA )*SIGMA
  ranev(idim) = raold(idim) +
( 2.0 * ranf ( dummy ) - 1.0 )*drmax
  ranev(idim) = ranev(idim) -
  dnint ( ranev(idim)/sigma )*sigma
  rbnew(idim) = rbold(idim) +
( 2.0 * ranf ( dummy ) - 1.0 )*drmax
  rbnew(idim) = rbnew(idim) -
  dnint ( rbnew(idim)/sigma )*sigma
  panev(idim) = paold(idim) +
( 2.0 * ranf ( dummy ) - 1.0 )*drmax
  pbnew(idim) = pbold(idim) +
( 2.0 * ranf ( dummy ) - 1.0 )*drmax
ENDDO

```

```

C ** CALCULATE THE ENERGY OF I IN THE NEW CONFIGURATION **

```

```

CALL PENERGY ( POTK, RXINEW, IP, C1,
VNEW, WNEW )

```

```

CALL CSKENERGY ( PANEV, PBNEW, IP, C1,
KENEW )

```

```

call cs(ranev,panev,rbnew,pbnew,
rxinev,rx(:,c1+1,ip),znew)
call cs(ranev,panev,rbnew,pbnew,
rx(:,c1-1,ip),rxinev,zn)
znew=znew*zn

```

```

C ** INSTANTANEOUS VALUE OF THE ACTION **

```

```

ACTNEW = KENEW + VNEW

```

```

DELTAV = VNEW - VOLD
DELTKE = KENEW - KEOLD

```

```

C ** CHECK FOR ACCEPTANCE **

```

```

DELTACTB = dble(znew)/dble(zold)*exp(-LS*(VNEW-VOLD))

```

```

IF ( DELTACTB .GT. RANF ( DUMMY ) ) THEN
  V = V + DELTAV
  KE = KE + DELTKE
  ACATMA = ACATMA + 1.0
  DO IDIM = 1, DIM

```

```

c imaginary time periodic boundary conditions

```

```

IF (C1.EQ.1) THEN
  RX(IDIM,FTNO+1,PIP)=RX(IDIM,C1,IP)
  RA(IDIM,FTNO+1,PIP)=RA(IDIM,C1,IP)
  RB(IDIM,FTNO+1,PIP)=RB(IDIM,C1,IP)
ENDIF

```

```

IF (C1.EQ.FTNO) THEN
  RX(IDIM,0,NIP)=RX(IDIM,C1,IP)
  RA(IDIM,0,NIP)=RA(IDIM,C1,IP)
  RB(IDIM,0,NIP)=RB(IDIM,C1,IP)
ENDIF

```

```

ENDDO

```

```

ENDIF

```

```

IF (STEP.GT.IEQUI) THEN
  ACM = ACM + 1.0

```

```

C ** CALCULATE INSTANTANEOUS VALUES **

```

```

IF (POTK .EQ. 'LJ') THEN
  VN = ( V + VLRC )
ELSE
  VN = V
ENDIF

```

```

C ** ACCUMULATE AVERAGES **

```

```

ACV = ACV + VN
ACVSQ = ACVSQ + VN*VN
ACKE = ACKE + KE
ACKESQ = ACKESQ + KE*KE
ENDF

```

```

C *****

```

```

C ** END DISPLACEMENT MOVE **

```

```

C *****

```

```

90 CONTINUE

```

```

C *****
C ** ENDS LOOP OVER TIME SLICES **
C *****
97 CONTINUE
C ** WRITE OUT THE INSTANTANEOUS VALUES ON FORT.9 **
  IF ( MOD ( STEP, IPRINT ) .EQ. 0 ) THEN
    WRITE(9,*) STEP, KE/FTNO, V/FTNO
  ENDIF
C ** CREATE POSITION DISTRIBUTION **
  CALL DISTR(30_8,STEP*FTNO*NP/30)
C ** PERFORM PERIODIC OPERATIONS **
  IF ( MOD ( STEP, IRATIO ) .EQ. 0 ) THEN
C ** ADJUST MAXIMUM DISPLACEMENT **
  RATIO = ACATMA / DBLE ( FTNO * IRATIO )
  RATIOB = ACATMAB / DBLE ( IRATIO )
  RATIOS = ACATMAS / DBLE ( IRATIO )
  IF ( RATIO .GT. 0.5 ) THEN
C DRMAX = DRMAX * 1.05
  ELSE
C DRMAX = DRMAX * 0.95
  ENDIF
  ACATMA = 0.0
  ACATMAB = 0.0
  ACATMAS = 0.0
  ENDIF
  IF ( MOD ( STEP, IPRINT ) .EQ. 0 ) THEN
C ** WRITE OUT RUNTIME INFORMATION **
  WRITE(*,*) ' step drmax'
  WRITE(*,*(18, 2XE10.4) ' ) STEP, DRMAX
  WRITE(*,*) ' acm ratio ratiob ratios
: ke v'
  WRITE(*,*(18,5(2XE10.4))' ) INT(ACM),
: RATIO, RATIOB, RATIOS,
: KE/FTNO, V/FTNO
  WRITE(*,*) ' <ke> <v>'
  WRITE(*,*(2(2XE13.7))' ) ACKE/ACM/FTNO, ACV/ACM/FTNO
  ENDIF
  IF ( MOD ( STEP, ISAVE ) .EQ. 0 ) THEN
C ** WRITE OUT THE CONFIGURATION AT INTERVALS **
  CALL WRITCN ( CNFILE )
  ENDIF
100 CONTINUE
C *****
C ** ENDS THE LOOP OVER CYCLES **
C *****
  WRITE(*,('(/' ' END OF MARKOV CHAIN ' '/' '))' )
C ** CHECKS FINAL VALUE OF THE POTENTIAL ENERGY IS CONSISTENT **
  CALL CSSUMUP (POTK, OVRLAP, KEEND, VEND)
  IF ( ABS(VEND - V) .GT. 1.0d-03 ) THEN
  WRITE(*,('( ' PROBLEM WITH V ENERGY !!! ' '))' )
  WRITE(*,('( ' VEND = ', E20.6) ' )' ) VEND
  WRITE(*,('( ' V = ', E20.6) ' )' ) V
  ENDIF
  IF ( ABS(KEEND - KE) .GT. 1.0d-03 ) THEN
  WRITE(*,('( ' PROBLEM WITH KE ENERGY !!! ' '))' )
  WRITE(*,('( ' KEEND = ', E20.6) ' )' ) KEEND
  WRITE(*,('( ' KE = ', E20.6) ' )' ) KE
  ENDIF
C ** WRITE OUT THE FINAL CONFIGURATION FROM THE RUN **
  CALL WRITCN ( CNFILE )
C ** CALCULATE AND WRITE OUT RUNNING AVERAGES **
  AVV = ACV / ACM
  ACVSQ = ( ACVSQ / ACM ) - AVV ** 2
  AVKE = ACKE / ACM
  ACKESQ = ( ACKESQ / ACM ) - AVKE ** 2
C ** CALCULATE FLUCTUATIONS **
  IF ( ACVSQ .GT. 0.0 ) FLV = SQRT ( ACVSQ/ACM )/FTNO
  IF ( ACKESQ .GT. 0.0 ) FLKE = SQRT ( ACKESQ/ACM )/FTNO
  WRITE(*,('( ' AVERAGES ' ' / ' '))' )
  WRITE(*,('( ' <V/N> = ', E12.6) ' )' ) AVV/FTNO
  WRITE(*,('( ' <KE/N> = ', E12.6) ' )' ) AVKE/FTNO
  WRITE(*,('( ' ' FLUCTUATIONS ' ' / ' '))' )
  WRITE(*,('( ' FLUCTUATION IN <V/N> = ', E12.6) ' )' ) FLV
  WRITE(*,('( ' FLUCTUATION IN <KE/N> = ', E12.6) ' )' ) FLKE
  WRITE(*,('( ' ' END OF SIMULATION ' ' / ' '))' )
C *****
C ** THE END **
C *****
  STOP
  END
  subroutine permcyc()
  implicit none
  ! given a particle i, lext(i) returns the cycle it belongs to
  INCLUDE 'mc-cs.par'
  integer*8 i,j,k,ii(mmp),jj,kk,ll
  kk=1
  do i=1,np
  ll=1
  ii(ll)=i
  ! jj=next(ii(ll))
  do j=1,ll
  if(jj.eq.ii(j))goto 2
  enddo
  ll=ll+1
  ii(ll)=jj
  goto 1
  ! do k=1,ll
  ! lext(ii(k))=kk
  enddo
  kk=kk+1
  enddo
  return
  end
  subroutine switch(i,j)
  ! switch i and j
  implicit none
  integer*8 i,j,k
  k=i
  i=j
  j=k
  return
  end
  subroutine update(j,rxp,rap,rbp,ip,nip)
  ! updates a portion of the current path x using the proposed path xp
  implicit none
  INCLUDE 'mc-cs.par'
  integer*8 ip,nip,j,l,k,imid
  integer*8 mcm
  real*8 rxp(mdim,0:n)
  real*8 rap(mdim,0:n),rbp(mdim,0:n)
  mcm = j+mbm-floor((j+mbm-.1)/ftn0)*ftn0
  l=0
  if(j+mbm.le.ftn0)then
  do k=j+1,mcm-1
  l=l+1
  do imid=1,dim
  rx(imid,k,ip)=rxp(imid,l)
  ra(imid,k,ip)=rap(imid,l)
  rb(imid,k,ip)=rbp(imid,l)
  enddo
  enddo
  else
  do k=j+1,ftn0
  l=l+1
  do imid=1,dim
  rx(imid,k,ip)=rxp(imid,l)
  ra(imid,k,ip)=rap(imid,l)
  rb(imid,k,ip)=rbp(imid,l)
  enddo
  enddo
  do k=1,mcm-1
  l=l+1
  do imid=1,dim
  rx(imid,k,nip)=rxp(imid,l)
  ra(imid,k,nip)=rap(imid,l)
  rb(imid,k,nip)=rbp(imid,l)
  enddo
  enddo
  do imid=1,dim
  rx(imid,ftn0+1,ip)=rx(imid,1,nip)
  rx(imid,0,nip)=rx(imid,ftn0,ip)
  ra(imid,ftn0+1,ip)=ra(imid,1,nip)
  ra(imid,0,nip)=ra(imid,ftn0,ip)
  rb(imid,ftn0+1,ip)=rb(imid,1,nip)
  rb(imid,0,nip)=rb(imid,ftn0,ip)
  enddo
  return
  end
  subroutine swap(j,ip,nip,kp,nkp)
  implicit none

```

```

! updates a portion of the current path x using the proposed path xp
INCLUDE      'mc-cs.par'

integer*8 ip,nip,kp,nkp,j,k,idim
integer*8 mcm
real*8 rr

mcm = j+mbm-floor((j+mbm-.1)/ftn0)*ftn0

if(j+mbm.le.ftn0)then
do k=mcm,ftn0
do idim=1,dim
rr=rx(idim,k,ip)
rx(idim,k,ip)=rx(idim,k,ip)
rx(idim,k,ip)=rr
rr=ra(idim,k,ip)
ra(idim,k,ip)=ra(idim,k,ip)
ra(idim,k,ip)=rr
rr=rb(idim,k,ip)
rb(idim,k,ip)=rb(idim,k,ip)
rb(idim,k,ip)=rr
rr=pa(idim,k,ip)
pa(idim,k,ip)=pa(idim,k,ip)
pa(idim,k,ip)=rr
rr=pb(idim,k,ip)
pb(idim,k,ip)=pb(idim,k,ip)
pb(idim,k,ip)=rr
enddo
enddo
do idim=1,dim
rx(idim,ftn0+1,ip)=rx(idim,1,nkp)
rx(idim,ftn0+1,ip)=rx(idim,1,nip)
rx(idim,0,nip)=rx(idim,ftn0,ip)
rx(idim,0,nkp)=rx(idim,ftn0,ip)
ra(idim,ftn0+1,ip)=ra(idim,1,nkp)
ra(idim,ftn0+1,ip)=ra(idim,1,nip)
ra(idim,0,nip)=ra(idim,ftn0,ip)
ra(idim,0,nkp)=ra(idim,ftn0,ip)
rb(idim,ftn0+1,ip)=rb(idim,1,nkp)
rb(idim,ftn0+1,ip)=rb(idim,1,nip)
rb(idim,0,nip)=rb(idim,ftn0,ip)
rb(idim,0,nkp)=rb(idim,ftn0,ip)
enddo
enddo
return
end

subroutine bridge(x0,x1,std,xnew,out)
implicit none
! sample n gaussians with std from xnew(0)=x0 to xnew(ftn0)=x1
INCLUDE      'mc-cs.par'

integer*8 l1,l2,l3,j,out,idim
real*8 std,d,s,xi
real*8 x0(mdim),x1(mdim),xnew(mdim,0:n)

out=0
l3=mbm
do idim=1,dim
xnew(idim,0)=x0(idim)
xnew(idim,l3)=x0(idim)+((x1(idim)-x0(idim))-
dnint((x1(idim)-x0(idim))/sigma)*sigma)
enddo
do j=1,mbm-1
l1=j-1
l2=j
s=std*(dble(l3-l2)/dble(l3-l1))*0.5d0
do idim=1,dim
d=xnew(idim,l3)-xnew(idim,l1)
d=d-dnint(d/sigma)*sigma
xnew(idim,j)=xnew(idim,l1)+d/dble(l3-l1)+xi(s)
xnew(idim,j)=xnew(idim,j)-dnint(xnew(idim,j)/sigma)*sigma
enddo
enddo
return
end

function xi(std)
! sample a gaussian with standard deviation std (box-muller method)
implicit none
real*8 xi,std,pi,ranf
data pi/3.14159265358979323846264338328d0/
xi=cos(pi*ranf(0.d0))*std*sqrt(-2.d0*log(tiny(pi)+ranf(0.d0)))
return
end

SUBROUTINE DISTR(NN,NORM)
IMPLICIT NONE
C WRITES ON FORT.10 THE X-POSITION DISTRIBUTION
INCLUDE      'mc-cs.par'

REAL*8 DS,DIST(0:1000)
INTEGER*8 NN,NORM,I,J,K
SAVE DIST

DS=SIGMA/NN

DO I=0,NN
DO J=1,FTNO
DO K=1,NP
IF(-SIGMA/2+(I-.5)*DS.LT.RX(1,J,K).AND.
RX(1,J,K).LT.-SIGMA/2+(I+.5)*DS) THEN
DIST(I)=DIST(I)+1.D0
ENDIF
ENDDO
ENDDO
WRITE(10,*) -SIGMA/2+I*DS,DIST(I)/NORM
ENDDO

```

```

CLOSE(UNIT=10)

```

```

RETURN
END

```

```

FUNCTION FACT ( N )
IMPLICIT NONE
C FACTORIAL FUNCTION
INTEGER*8 FACT,N,P,I
P=1
DO I=1,N
P=P*I
ENDDO
FACT=P
END

```

```

SUBROUTINE CSSUMUP (POTK, OVLAP, KE, V)
IMPLICIT NONE

```

```

C *****
C ** CALCULATES THE TOTAL ACTION **
C ** USAGE: **
C ** THE SUBROUTINE RETURNS THE TOTAL ACTION AT THE **
C ** BEGINNING AND END OF THE RUN. **
C *****

```

```

INCLUDE      'mc-cs.par'

```

```

REAL*8 V, KE, VV, KK
LOGICAL OVLAP
CHARACTER POTK*(*)

```

```

REAL*8 RXII, RXIJ
REAL*8 VIJ, WIJ, RIJSQ, W, WW

```

```

INTEGER*8 TAU, I, J, IDIM

```

```

C *****
C POTENTIAL ACTION

```

```

VV = 0.0

```

```

C ** LOOP OVER ALL THE PAIRS IN THE LIQUID **

```

```

DO TAU = 1, FTNO
DO 100 I = 1, NP - 1
DO 99 J = I + 1, NP
RIJSQ = 0.d0
DO IDIM = 1, DIM
RXIJ = RX(IDIM,TAU,I) - RX(IDIM,TAU,J)

```

```

C ** MINIMUM IMAGE THE PAIR SEPARATIONS **
RXIJ = RXIJ -
DNINT ( RXIJ/SIGMA ) * SIGMA
RIJSQ = RIJSQ + RXIJ * RXIJ
ENDDO

```

```

CALL POT (RIJSQ, VIJ, WIJ, POTK)

```

```

VV = VV + VIJ

```

```

C
99 WW = WW + WIJ

```

```

100 CONTINUE

```

```

ENDDO

```

```

C KINETIC ACTION

```

```

KK = 0.0

```

```

DO TAU = 1, FTNO
DO I = 1, NP
DO IDIM = 1, DIM
KK=KK+(PA(IDIM,TAU,I)**2.+PB(IDIM,TAU,I)**2.)/FTM
ENDDO
ENDDO

```

```

ENDDO

```

```

KE=KK

```

```

ENDDO

```

```

RETURN

```

```

END

```

```

SUBROUTINE PENERGY ( POTK, RXI, I, TAU,

```

```

: V, W )

```

```

IMPLICIT NONE

```

```

C *****
C ** RETURNS THE POTENTIAL ENERGY OF ATOM I WITH ALL OTHER ATOMS. **
C ** PRINCIPAL VARIABLES: **
C ** **

```

```

C ** INTEGER I THE ATOM OF INTEREST **
C ** INTEGER NP THE NUMBER OF ATOMS **
C ** INTEGER TAU THE TIMESTEP **

```

```

C ** REAL*8 RX, RY, RZ THE ATOM POSITIONS **
C ** REAL*8 RXI,RYI,RZI THE COORDINATES OF ATOM I **
C ** REAL*8 V THE POTENTIAL ENERGY OF ATOM I **
C ** REAL*8 W THE VIRIAL OF ATOM I **

```

```

C ** **
C ** USAGE: **
C ** **

```

```

C ** THIS SUBROUTINE IS USED TO CALCULATE THE CHANGE OF ENERGY **
C ** DURING A TRIAL MOVE OF ATOM I. IT IS CALLED BEFORE AND **
C ** AFTER THE RANDOM DISPLACEMENT OF I. **
C *****

```

```

INCLUDE      'mc-cs.par'

```

```

REAL*8 RXI(MDIM), V, W

```

```

INTEGER*8 I, J, TAU, IDIM

```

```

CHARACTER POTK*(*)

```

```

REAL*8      RXIJ, RIJSQ, VIJ, WIJ
C *****
V      = 0.0
W      = 0.0
C ** LOOP OVER ALL MOLECULES EXCEPT I **
DO 100 J = 1, NP
  IF ( I .NE. J ) THEN
    RIJSQ = 0.d0
    DO IDIM = 1, DIM
      RXIJ = RXI(IDIM) - RX(IDIM,TAU,J)
      RXIJ = RXIJ -
      : DNINT ( RXIJ/SIGMA ) * SIGMA
      RIJSQ = RIJSQ + RXIJ * RXIJ
    ENDDO
    CALL POT (RIJSQ, VIJ, WIJ, POTK)
    V = V + VIJ
    W = W + WIJ
  90  ENDIF
100  CONTINUE
RETURN
END

SUBROUTINE PPENERGY ( POTK, RXI, I, RXJ, J,
:  TAU, V, W )
IMPLICIT NONE
C *****
C ** RETURNS THE POTENTIAL ENERGY OF ATOMS I AND J WITH **
C ** ALL OTHER ATOMS PLUS THE ONE BETWEEN I AND J **
C ** PRINCIPAL VARIABLES: **
C ** **
C ** INTEGER I, J          THE ATOMS OF INTEREST **
C ** INTEGER NP           THE NUMBER OF ATOMS **
C ** INTEGER TAU          THE TIMESTEP **
C ** REAL*8  RX, RY, RZ   THE ATOM POSITIONS **
C ** REAL*8  RXI,RYI,RZI  THE COORDINATES OF ATOM I **
C ** REAL*8  RXJ,RYJ,RZJ  THE COORDINATES OF ATOM J **
C ** REAL*8  V             THE POTENTIAL ENERGY OF ATOM I **
C ** REAL*8  W             THE VIRIAL OF ATOM I **
C ** **
C ** USAGE: **
C ** **
C ** THIS SUBROUTINE IS USED TO CALCULATE THE CHANGE OF ENERGY **
C ** DURING A TRIAL MOVE OF ATOM I. IT IS CALLED BEFORE AND **
C ** AFTER THE RANDOM DISPLACEMENT OF I. **
C *****
INCLUDE 'mc-cs.par'
REAL*8  RXI(MDIM), RXJ(MDIM), V, W
INTEGER*8 I, J, K, TAU, IDIM
CHARACTER  POTK(*)
REAL*8  RXIJ, RIJSQ, VIJ, WIJ
C *****
V      = 0.0
W      = 0.0
C ** LOOP OVER ALL MOLECULES EXCEPT I AND J **
DO 100 K = 1, NP
  IF ( K .NE. I .AND. K .NE. J ) THEN
    RIJSQ = 0.d0
    DO IDIM = 1, DIM
      RXIJ = RXI(IDIM) - RX(IDIM,TAU,K)
      RXIJ = RXIJ -
      : DNINT ( RXIJ/SIGMA ) * SIGMA
      RIJSQ = RIJSQ + RXIJ * RXIJ
    ENDDO
    CALL POT (RIJSQ, VIJ, WIJ, POTK)
    V = V + VIJ
    W = W + WIJ
  IF ( I .NE. J ) THEN
    RIJSQ = 0.d0
    DO IDIM = 1, DIM
      RXIJ = RXJ(IDIM) - RX(IDIM,TAU,K)
      RXIJ = RXIJ -
      : DNINT ( RXIJ/SIGMA ) * SIGMA
      RIJSQ = RIJSQ + RXIJ * RXIJ
    ENDDO
    CALL POT (RIJSQ, VIJ, WIJ, POTK)
    V = V + VIJ
    W = W + WIJ
  ENDIF
  ENDIF
100  CONTINUE
C RETURN
IF ( I .NE. J ) THEN
  RIJSQ = 0.d0
  DO IDIM = 1, DIM
    RXIJ = RXI(IDIM) - RXJ(IDIM)
    RXIJ = RXIJ -
    : DNINT ( RXIJ/SIGMA ) * SIGMA
    RIJSQ = RIJSQ + RXIJ * RXIJ
  ENDDO
  CALL POT (RIJSQ, VIJ, WIJ, POTK)
  V = V + VIJ
  W = W + WIJ
ENDIF
RETURN
END

SUBROUTINE CSKENERGY ( PAI, PBI, I, TAU, KE )
IMPLICIT NONE
C *****
C ** RETURNS THE KINETIC ENERGY OF ATOM I WITH ALL OTHER ATOMS. **
C ** PRINCIPAL VARIABLES: **
C ** **
C ** INTEGER I          THE ATOM OF INTEREST **
C ** INTEGER NP         THE NUMBER OF ATOMS **
C ** INTEGER TAU        THE TIMESTEP **
C ** REAL*8  RX, RY, RZ  THE ATOM POSITIONS **
C ** REAL*8  RXI,RYI,RZI THE COORDINATES OF ATOM I **
C ** REAL*8  KE          THE KINETIC ENERGY OF ATOM I **
C ** **
C ** USAGE: **
C ** **
C ** THIS SUBROUTINE IS USED TO CALCULATE THE CHANGE OF ENERGY **
C ** DURING A TRIAL MOVE OF ATOM I. IT IS CALLED BEFORE AND **
C ** AFTER THE RANDOM DISPLACEMENT OF I. **
C *****
INCLUDE 'mc-cs.par'
REAL*8  PAI(MDIM), PBI(MDIM), KE
INTEGER*8 I, TAU, IDIM
C *****
KE = 0.d0
DO IDIM = 1, DIM
  KE=KE+(PAI(IDIM)**2.+PBI(IDIM)**2.)/4/FTM
ENDDO
RETURN
END

SUBROUTINE CSKKNENERGY ( I, TAU, KE )
IMPLICIT NONE
C *****
C ** RETURNS THE KINETIC ENERGY OF ATOM I WITH ALL OTHER ATOMS. **
C ** PRINCIPAL VARIABLES: **
C ** **
C ** INTEGER I          THE ATOM OF INTEREST **
C ** INTEGER NP         THE NUMBER OF ATOMS **
C ** INTEGER TAU        THE TIMESTEP **
C ** REAL*8  RX, RY, RZ  THE ATOM POSITIONS **
C ** REAL*8  RXI,RYI,RZI THE COORDINATES OF ATOM I **
C ** REAL*8  KE          THE KINETIC ENERGY OF ATOM I **
C ** **
C ** USAGE: **
C ** **
C ** THIS SUBROUTINE IS USED TO CALCULATE THE CHANGE OF ENERGY **
C ** DURING A TRIAL MOVE OF ATOM I. IT IS CALLED BEFORE AND **
C ** AFTER THE RANDOM DISPLACEMENT OF I. **
C *****
INCLUDE 'mc-cs.par'
REAL*8  KE
INTEGER*8 I, NI, TAU, IDIM
C *****
KE = 0.d0
DO IDIM = 1, DIM
  KE=KE+(PA(IDIM,TAU,I)**2.+PB(IDIM,TAU,I)**2.)/4/FTM
ENDDO
RETURN
END

REAL*8 FUNCTION RANF ( DUMMY )
C *****
C ** RETURNS A UNIFORM RANDOM VARIATE IN THE RANGE 0 TO 1. **
C ** **
C ** ***** **
C ** ** WARNING **
C ** ***** **
C ** **
C ** GOOD RANDOM NUMBER GENERATORS ARE MACHINE SPECIFIC. **
C ** PLEASE USE THE ONE RECOMMENDED FOR YOUR MACHINE. **
C ** **
C ** RAND(FLAG) returns a pseudo-random number from a uniform **
C ** distribution between 0 and 1. If FLAG is 0, the next number **
C ** in the current sequence is returned; if FLAG is 1, the **

```

```

C ** generator is restarted by CALL SRAND(0); if FLAG has any **
C ** other value, it is used as a new seed with SRAND. **
C *****
REAL*8 DUMMY

RANF = RAND ( )

RETURN
END

SUBROUTINE READCN ( CNFILE )
IMPLICIT NONE
C *****
C ** SUBROUTINE TO READ IN THE CONFIGURATION FROM UNIT 10 **
C *****
INCLUDE 'mc-cs.par'

CHARACTER CNFILE*(*) , HASH*1, MYFMT*77

INTEGER*8 CNUNIT, I, J, NNP, NI, IDIM
PARAMETER ( CNUNIT = 10 )

C *****

OPEN ( UNIT = CNUNIT, FILE = CNFILE, STATUS = 'OLD' )
WRITE(MYFMT, '(A,I10,A)') '(I3,3X, ',5*DIM, '(F12.6,3X))'

READ ( CNUNIT,* ) HASH, FTNO, NNP
IF ( NNP .NE. NP ) STOP 'N ERROR IN READCN'

DO 100 I = 1, NNP
  READ ( CNUNIT,* ) HASH, NEXT(I)
  NI = NEXT(I)
  DO 90 J = 1, FTNO+1
    READ ( CNUNIT, MYFMT ) LEXT(I), (RX(IDIM,J,I),IDIM=1,DIM),
: (RA(IDIM,J,I),IDIM=1,DIM), (RB(IDIM,J,I),IDIM=1,DIM),
: (PA(IDIM,J,I),IDIM=1,DIM), (PB(IDIM,J,I),IDIM=1,DIM)
90  ENDDO
  READ ( CNUNIT,13 )
  READ ( CNUNIT,13 )
  DO IDIM = 1, DIM
    RX(IDIM,0,NI) = RX(IDIM,FTNO,I)
100  ENDDO
13  FORMAT(A2)

CLOSE ( UNIT = CNUNIT )

RETURN
END

SUBROUTINE WRITCN ( CNFILE )
IMPLICIT NONE
C *****
C ** SUBROUTINE TO WRITE OUT THE CONFIGURATION TO UNIT 10 **
C *****
INCLUDE 'mc-cs.par'

CHARACTER CNFILE*(*) , MYFMT*77

REAL*8 RXI(DIM), RAI(DIM), RBI(DIM), PAI(DIM), PBI(DIM)
INTEGER*8 CNUNIT, I, J, IDIM
PARAMETER ( CNUNIT = 10 )

C *****

OPEN ( UNIT = CNUNIT, FILE = CNFILE, STATUS = 'UNKNOWN' )
WRITE(MYFMT, '(A,I10,A)') '(I3,3X, ',5*DIM, '(F12.6,3X))'

WRITE(*,*) 'output to file -----'
:-----
CALL FLUSH ( CNUNIT )
WRITE ( CNUNIT,* ) '#',FTNO,NP

DO 100 I = 1, NP
  WRITE ( CNUNIT,* ) '#', NEXT(I)
  DO 90 J = 1, FTNO+1
    DO IDIM = 1, DIM
      RXI (IDIM) = RX (IDIM,J,I)
      RAI (IDIM) = RA (IDIM,J,I)
      RBI (IDIM) = RB (IDIM,J,I)
      PAI (IDIM) = PA (IDIM,J,I)
      PBI (IDIM) = PB (IDIM,J,I)
    ENDDO
    WRITE (CNUNIT, MYFMT) LEXT(I), (RXI (IDIM),IDIM=1,DIM),
: (RAI (IDIM),IDIM=1,DIM), (RBI (IDIM),IDIM=1,DIM),
: (PAI (IDIM),IDIM=1,DIM), (PBI (IDIM),IDIM=1,DIM)
90  ENDDO
  WRITE ( CNUNIT,13 ) ''
  WRITE ( CNUNIT,13 ) ''
100  ENDDO
13  FORMAT(A2)

CLOSE ( UNIT = CNUNIT )

RETURN
END

SUBROUTINE INITCN ( CNFILE )
IMPLICIT NONE
C *****
C ** SUBROUTINE TO INITIALIZE THE CONFIGURATION TO UNIT 10 **
C *****
INCLUDE 'mc-cs.par'

REAL*8 RANF
CHARACTER CNFILE*(*) , MYFMT*77

INTEGER*8 CNUNIT, I, J, IDIM, I1
REAL*8 RXI(DIM), RXII(DIM, NP), RXIJ, RIJSQ
PARAMETER ( CNUNIT = 10 )

C *****

OPEN ( UNIT = CNUNIT, FILE = CNFILE, STATUS = 'UNKNOWN' )
WRITE(MYFMT, '(A,I10,A)') '(I3,3X, ',5*DIM, '(F12.6,3X))'

WRITE ( CNUNIT,* ) '#',FTNO,NP
RXII = 0.d0

DO 100 I = 1, NP
  WRITE ( CNUNIT,* ) '#', NEXT(I)
  DO IDIM = 1, DIM
    RXI (IDIM) = SIGMA*(RANF(0.d0)-.5)
  ENDDO
  DO I1 = 1, NP
    RIJSQ = 0.d0
    DO IDIM = 1, DIM
      RXIJ = RXI (IDIM)-RXII (IDIM,I1)
      RXIJ = RXIJ -
: DHINT ( RXIJ/SIGMA ) *SIGMA
      RIJSQ = RIJSQ + RXIJ * RXIJ
    ENDDO
    IF (RIJSQ.LE.EPSR+EPSR) THEN
      GOTO 10
    ENDIF
  ENDDO
  DO IDIM = 1, DIM
    RXII (IDIM,I) = RXI (IDIM)
  ENDDO
  DO J = 1, FTNO+1
    WRITE (CNUNIT, MYFMT) I, (RXI (IDIM),IDIM=1,DIM),
: (RXI (IDIM),IDIM=1,DIM), (RXI (IDIM),IDIM=1,DIM),
: (RXI (IDIM),IDIM=1,DIM), (RXI (IDIM),IDIM=1,DIM)
  ENDDO
  WRITE ( CNUNIT,13 ) ''
  WRITE ( CNUNIT,13 ) ''
100  CONTINUE
13  FORMAT(A2)

CLOSE ( UNIT = CNUNIT )

RETURN
END

SUBROUTINE POT ( RIJSQ, VIJ, WIJ, POTK )
IMPLICIT NONE
C *****
C ** SUBROUTINE FOR THE PAIR POTENTIAL **
C *****
INCLUDE 'mc-cs.par'

CHARACTER POTK*(*)
REAL*8 RIJ, RIJSQ, RMIN, RMSQ, RCSQ, SR2, SR6, VIJ, WIJ
REAL*8 COU3, F
PARAMETER ( COU3 = 4.76015472795910701328763470057d0 )

C *****
PI = ACOS(-1.d0)

VIJ = 0.0
WIJ = 0.0
IF (DIM .GT. 3) RETURN
IF (POTK .EQ. 'FREE') RETURN

IF (POTK .EQ. 'LJ') THEN
C *****
C *****
C *****
C *****
RMIN = 2.d0**(1./6)
RMIN = TINY(PI)
RMIN = EPSA
RMIN = 0.d0
RIJSQ = RIJSQ / (SIG * SIG)
RMSQ = RMIN*RMIN
RCSQ = RCUT*RCUT
IF (RIJSQ .LT. RMSQ) THEN
  RIJSQ = RMSQ
  SR2 = 1.d0 / RIJSQ
  SR6 = SR2 * SR2 * SR2
  VIJ = SR6 * ( SR6 - 1.d0 )
  WIJ = SR6 * ( SR6 - 5.d-1 )
  VIJ = 4.d0 * EPS * VIJ
  WIJ = 48.d0 * EPS * VIJ / 3.d0
ELSEIF (RIJSQ .GT. RMSQ .AND. RIJSQ .LT. RCSQ) THEN
  SR2 = 1.d0 / RIJSQ
  SR6 = SR2 * SR2 * SR2
  VIJ = SR6 * ( SR6 - 1.d0 )
  WIJ = SR6 * ( SR6 - 5.d-1 )
  VIJ = 4.d0 * EPS * VIJ
  WIJ = 48.d0 * EPS * VIJ / 3.d0
ENDIF
ELSEIF (POTK .EQ. 'BUMP') THEN
  RIJ = SQRT( RIJSQ )
  IF (RIJ .LE. SIG) THEN
    VIJ = EPSR
  ENDF
ELSEIF (POTK .EQ. 'PSW') THEN
  RIJ = SQRT( RIJSQ )
  IF (RIJ .LE. SIG) THEN
    VIJ = EPSR
  ELSEIF (RIJ .LE. SIG + RCUT .AND. RIJ .GT. SIG) THEN
    VIJ = -EPSA
  ENDF
ELSEIF (POTK .EQ. 'COULOMB') THEN
  F = NP/(NP-1)
  RIJ = SQRT( RIJSQ )
  IF (RIJ .GT. EPS) THEN
    IF (DIM .EQ. 3) THEN
      VIJ = -F*COU3*SIG/SIGMA/2.
      VIJ = VIJ * SIG/RIJ
    ELSEIF (DIM .EQ. 2) THEN

```

```

        VIJ = -F*(6.-PI*LOG(4.))-4.*LOG(SIGMA/SIG)/4.
        VIJ = VIJ - LOG(RIJ/SIG)
    ELSEIF (DIM .EQ. 1) THEN
        VIJ = F*SIGMA/SIG/4.
        VIJ = VIJ - RIJ/SIG
    ENDIF
ELSE
    RIJ = EPS
    IF (DIM .EQ. 3) THEN
        VIJ = -F*COU3*SIG/SIGMA/2.
        VIJ = VIJ + SIG/RIJ
    ELSEIF (DIM .EQ. 2) THEN
        VIJ = -F*(6.-PI*LOG(4.))-4.*LOG(SIGMA/SIG)/4.
        VIJ = VIJ - LOG(RIJ/SIG)
    ELSEIF (DIM .EQ. 1) THEN
        VIJ = F*SIGMA/SIG/4.
        VIJ = VIJ - RIJ/SIG
    ENDIF
ENDIF
ENDIF
ELSEIF (POTK .EQ. 'HARMONIC') THEN
    VIJ = 0.5*RIJSQ/SIG**2.
ENDIF

RETURN
END

subroutine cs(qqa,ppa,qqb,ppb,q,qp,zeta)
implicit none
C *****
C ** THE CHOERENT STATE
C *****
INCLUDE 'mc-cs.par'

complex*16 ii,aa(mdim),bb(mdim),psia,psib,gab,zeta
real*8 mo,qqa(mdim),ppa(mdim),qqb(mdim),ppb(mdim)
real*8 q(mdim),qp(mdim),aux
real*8 rgab,rzeta
integer*8 idim
parameter ( ii = cmplx(0.d0,1.d0) )

c harmonic oscillator
mo = moho

do idim=1,dim
aa(idim) = (mo*qqa(idim)+ii*ppa(idim))/sqrt(2*mo)
bb(idim) = (mo*qqb(idim)+ii*ppb(idim))/sqrt(2*mo)
enddo

c coherent states
psia=0.d0
psib=0.d0
do idim=1,dim
aux=q(idim)-sqrt(2/mo)*dble(aa(idim))
aux=aux-dnint(aux/sigma)*sigma
psia = psia - aux**2.*mo/2
psia = psia + ii*aux*sqrt(2*mo)*dimag(aa(idim))

aux=qp(idim)-sqrt(2/mo)*dble(bb(idim))
aux=aux-dnint(aux/sigma)*sigma
psib = psib - aux**2.*mo/2
psib = psib + ii*aux*sqrt(2*mo)*dimag(bb(idim))
enddo
psia = exp(psia)
psia = (mo/pi)**(dim/4.)*psia

psib = exp(psib)
psib = (mo/pi)**(dim/4.)*psib

c normalization
gab = 0.d0
do idim=1,dim
gab = gab - (cdabs(aa(idim))**2.+cdabs(bb(idim))**2.)/2
gab = gab + dconjg(aa(idim))*bb(idim)
gab = gab + (qqa(idim)*ppa(idim)-qqb(idim)*ppb(idim))*ii/2
enddo
gab = exp(gab)
rgab = dble(gab)

c zeta
zeta = 0.d0
do idim=1,dim
zeta = zeta - 1s*(ppb(idim)**2.+ppa(idim)**2.)/4/ftm
enddo
zeta = psia*dconjg(psib)*gab*exp(zeta)
rzeta = dble(zeta)

c print *, zeta

return
end

subroutine update_cs(j,rxp,rap,rbp,pap,pbp,ip,nip)
implicit none
! updates a portion of the current path x using the proposed path xp
INCLUDE 'mc-cs.par'

integer*8 ip,nip,j,l,k,idim
real*8 rxp(mdim,0:n)
real*8 rap(mdim,0:n),rbp(mdim,0:n)
real*8 pap(mdim,0:n),pbp(mdim,0:n)

l=0
do k=j+1,ftn0
l=l+1
do idim=1,dim
rx(idim,k,ip)=rxp(idim,l)
ra(idim,k,ip)=rap(idim,l)
rb(idim,k,ip)=rbp(idim,l)
pa(idim,k,ip)=pap(idim,l)

pb(idim,k,ip)=pbp(idim,l)
enddo
enddo
do k=1,j-1
l=l+1
do idim=1,dim
rx(idim,k,nip)=rxp(idim,l)
ra(idim,k,nip)=rap(idim,l)
rb(idim,k,nip)=rbp(idim,l)
pa(idim,k,nip)=pap(idim,l)
pb(idim,k,nip)=pbp(idim,l)
enddo
enddo
do idim=1,dim
rx(idim,ftn0+1,ip)=rx(idim,1,nip)
rx(idim,0,nip)=rx(idim,ftn0,ip)
enddo

return
end

subroutine acccs(p,ip,kp,j,rxp,rxpp,rap,rapp,rbp,rbpp)
implicit none
! complete acceptance probability
INCLUDE 'mc-cs.par'

integer*8 ip,kp,j,idim
real*8 rxp(mdim,0:n),rxpp(mdim,0:n)
real*8 rap(mdim,0:n),rapp(mdim,0:n)
real*8 rbp(mdim,0:n),rbpp(mdim,0:n)
real*8 p,rho,rrx
integer*8 mcm,i,ll,nip,nkp

mcm = j+mbs-floor((j+mbs-.1)/ftn0)*ftn0
nip=next(ip)
nkp=next(kp)

rho=0.d0
ll=-1
if(j+mbs.le.ftn0)then
do i=j,mcm-1
ll=ll+1
do idim=1,dim
rrx=rxp(idim,ll)-rxp(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2
rrx=rap(idim,ll)-rap(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2
rrx=rbp(idim,ll)-rbp(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2

rrx=rx(idim,i,ip)-rx(idim,i+1,ip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
rrx=ra(idim,i,ip)-ra(idim,i+1,ip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
rrx=rb(idim,i,ip)-rb(idim,i+1,ip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
enddo
enddo
else
do i=j,ftn0
ll=ll+1
do idim=1,dim
rrx=rxp(idim,ll)-rxp(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2
rrx=rap(idim,ll)-rap(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2
rrx=rbp(idim,ll)-rbp(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2

rrx=rx(idim,i,ip)-rx(idim,i+1,ip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
rrx=ra(idim,i,ip)-ra(idim,i+1,ip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
rrx=rb(idim,i,ip)-rb(idim,i+1,ip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
enddo
enddo
do i=1,mcm-1
ll=ll+1
do idim=1,dim
rrx=rxp(idim,ll)-rxp(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2
rrx=rap(idim,ll)-rap(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2
rrx=rbp(idim,ll)-rbp(idim,ll+1)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho+rrx**2

rrx=rx(idim,i,nip)-rx(idim,i+1,nip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
rrx=ra(idim,i,nip)-ra(idim,i+1,nip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
rrx=rb(idim,i,nip)-rb(idim,i+1,nip)
rrx=rrx-dnint(rrx/sigma)*sigma
rho=rho-rrx**2
enddo
enddo

```

```

      rho=rho-rrx**2
    enddo
  endif
  if (ip.ne.kp) then
    ll=1
    if (j+mbm.le.ftn0) then
      do i=j,mcm-1
        ll=ll+1
        do idim=1,dim
          rrx=rxpp(idim,ll)-rxpp(idim,ll+1)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho+rrx**2
          rrx=rapp(idim,ll)-rapp(idim,ll+1)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho+rrx**2
          rrx=rbpp(idim,ll)-rbpp(idim,ll+1)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho+rrx**2

          rrx=rx(idim,i,kp)-rx(idim,i+1,kp)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho-rrx**2
          rrx=ra(idim,i,kp)-ra(idim,i+1,kp)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho-rrx**2
          rrx=rb(idim,i,kp)-rb(idim,i+1,kp)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho-rrx**2
        enddo
      enddo
    else
      do i=j,ftn0
        ll=ll+1
        do idim=1,dim
          rrx=rxpp(idim,ll)-rxpp(idim,ll+1)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho+rrx**2
          rrx=rapp(idim,ll)-rapp(idim,ll+1)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho+rrx**2
          rrx=rbpp(idim,ll)-rbpp(idim,ll+1)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho+rrx**2

          rrx=rx(idim,i,kp)-rx(idim,i+1,kp)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho-rrx**2
          rrx=ra(idim,i,kp)-ra(idim,i+1,kp)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho-rrx**2
          rrx=rb(idim,i,kp)-rb(idim,i+1,kp)
          rrx=rrx-dnint(rrx/sigma)*sigma
          rho=rho-rrx**2
        enddo
      enddo
    do i=1,mcm-1
      ll=ll+1
      do idim=1,dim
        rrx=rxpp(idim,ll)-rxpp(idim,ll+1)
        rrx=rrx-dnint(rrx/sigma)*sigma
        rho=rho+rrx**2
        rrx=rapp(idim,ll)-rapp(idim,ll+1)
        rrx=rrx-dnint(rrx/sigma)*sigma
        rho=rho+rrx**2
        rrx=rbpp(idim,ll)-rbpp(idim,ll+1)
        rrx=rrx-dnint(rrx/sigma)*sigma
        rho=rho+rrx**2

        rrx=rx(idim,i,nkp)-rx(idim,i+1,nkp)
        rrx=rrx-dnint(rrx/sigma)*sigma
        rho=rho-rrx**2
        rrx=ra(idim,i,nkp)-ra(idim,i+1,nkp)
        rrx=rrx-dnint(rrx/sigma)*sigma
        rho=rho-rrx**2
        rrx=rb(idim,i,nkp)-rb(idim,i+1,nkp)
        rrx=rrx-dnint(rrx/sigma)*sigma
        rho=rho-rrx**2
      enddo
    enddo
  endif
  rho=ftm*rho/(2.*1s)
  p=exp(-rho)
  return
end

```

```

-----
mc-cs.par
-----
C *****

```

```

C ** MC-CS.PAR **
C *****
INTEGER*8 MDIM, N, MNP
REAL*8 PI

PARAMETER ( MDIM = 10 ) ! maximum number of dimensions
PARAMETER ( MNP = 1000 ) ! maximum number of particles
PARAMETER ( N = 3000 ) ! maximum number of time slices

PARAMETER ( PI = 3.14159265358979323846264338328d0 )

REAL*8 RX(MDIM,0:N,MNP)
REAL*8 RA(MDIM,0:N,MNP), RB(MDIM,0:N,MNP)
REAL*8 PA(MDIM,0:N,MNP), PB(MDIM,0:N,MNP)
REAL*8 FTM, LS
REAL*8 SIGMA, SIG, EPSA, EPSR, EPS, RCUT
REAL*8 MOHO
INTEGER*8 FTNO, NP, NEXT(MNP), LEXT(MNP), MEXT(N,MNP), MBM
INTEGER*8 DIM

! ghost path
COMMON / BLOCK0 / RA, RB, PA, PB
! real path
COMMON / BLOCK1 / RX, DIM
! pair-potential parameters
COMMON / BLOCK2 / SIG, EPSR, EPSA, EPS, RCUT
! mass, timestep, box edge, # timeslices, # particles
COMMON / BLOCK3 / FTM, LS, SIGMA, FTNO, NP
! permutations
COMMON / BLOCK4 / NEXT, LEXT, MEXT, MBM
! harmonic oscillator for coherent states
COMMON / BLOCK5 / MOHO

-----
data-cs-new.in
-----
0 number of spatial dimensions (1,2,3,...<= MDIM)
2
1 seed of the random sequence RAND
44
2 if bose (T/F)
T
3 number of particles (<= MNP)
16
4 the potential SUBROUTINE POT
LJ
5 # time slices = 1/temperature/timestep (< N)
250
6 mass (hbar = kb = 1)
0.0830594d0
7 # of cycles (nstep)
5000000000000000
8 # of steps between output lines (iprint)
100
9 # of steps between configuration saves (isave)
100
10 # of steps for equilibration (iequi)
0
11 # of steps for acceptance ratios (iratio)
100
12 configuration file name
conf.xyz
13 enter 0 if initialization needed
0
14 density THERMODYNAMICS
.05d0
15 temperature THERMODYNAMICS
.2d0
16 maximum displacement/box edge
.01d0
17 number of bridge timeslices (>= 1; <= #5)
50
18 potential cutoff distance (LJ) SUBROUTINE POT
2.5d0
19 sig (LJ 2.556) SUBROUTINE POT
2.556d0
20 epsr (LJ INIT) SUBROUTINE POT
3.d0
21 epsa SUBROUTINE POT
1.d0
22 eps (LJ 10.22) SUBROUTINE POT
1.022d1
23 moho = mass_ho*omega HO
5.814d0
24 if only displace (T/F)
F
25 if only bridge (T/F)
F
26 if zero path initially (0 in 13) (T/F)
F

```