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The Number Zero is Welcomed by Mathematics, but Physics Should Be Careful When Dealing with It

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Abstract
The number zero is happily used by mathematics, but not physics for various reasons. It should be incorrect to say that $\phi(x) = 0 = \gamma(x)$, when there are two, different, physical meanings, such as fields for two very different kinds of particles, etc. That could lead to some kind of confusion in physics. In that case, it could even be necessary to reject using the zero number in certain expressions, and instead, at least try to 'go around them'. Do not forget that the number zero was banned for 1,500 years, long, long ago!

Keywords
The Number Zero, Quantum Field Theory, Affine Quantization, Canonical Quantization, ϕ^4 Theory

Introduction

There have been many models with the same 'disease' as that of φ^4 [1, 2]. The secret to a valid canonical quantization is remarkably simple. All you need is the addition of a single, fixed potential, which is not seen in the texts, but it puts things in proper position elsewhere. This single, additional, potential is just $2\hbar^2/\varphi(x)^2$, alongside $\hat{\pi}(x)^2$. It is noteworthy that these potential forces $0 < |\varphi(x)| < \infty$ which leads to $0 < |\varphi(x)|^p < \infty$, and guarantees that almost all other potentials remain finite.

The secret to this magic has come from affine quantization [3].

This example would consider integrations in which $\int_{-1}^{1} |x|^p\, dx < \infty$ with $-1 < p < \infty$, would still get an identical result without needing $x = 0$. More importantly, the integral, $\int_{-1}^{1} |\varphi(x)|^p\, d\varphi(x)$ gets an identical result without needing $\varphi(x) = 0$.

Why Removing Certain Zeros can be Important

Suppose you were dealing with two very different fields, $\varphi(x)$ and $\beta(x)$. Is the mathematical equation $\varphi(x) = 0 = \beta(x)$ physically correct? To satisfy both mathematics and physics, it would be far more correct if such 'zero equations' are fully eliminated by removing them altogether.

Not only would that be mathematically acceptable, but it could have a positive effect on other equations, such as $\int e^{-A/x^2 - Bx^2}$ dx, which permits $x \neq 0$, or also said as $0 < |x| < \infty$. Of greater inter $\int e^{-C/\varphi(x)^2 - D\varphi(x)^2}\, d\varphi(x)$, which permits removing $\varphi(x) = 0$, or also said as $0 < |\varphi(x)| < \infty$.

How Removing a Particular Zero can Solve a Familiar Problem in Quantum Field Theory

The familiar φ^4 model, using canonical quantization, which did not get positive results stems from

$$\mathcal{H} = \int \{ \tfrac{1}{2}[\hat{\pi}(x)^2 + (\overrightarrow{\nabla}\varphi(x))^2 + m^2\varphi(x)^2] + g\,\varphi(x)^4 \}\, d^3x, \tag{2.1}$$

which leads to the fact that $g > 0$ was in the equation to be solved, but the results were as if $g = 0$. Now, using affine quantization, it has been able to correctly solve that equation with the presence of a new $\hbar$-term, specifically $2\hbar^2/\varphi(x)^2$, added just after the kinetic term in the quantum Hamiltonian, which allows g to do its job correctly.

Observe that the added $\hbar$-term was available because it required that $\varphi(x) \neq 0$. Now using standard quantum variables has lead us to

$$\mathcal{H} = \int\{\tfrac{1}{2}[\hat{\pi}(x)^2 + 2\hbar^2/\varphi(x)^2 + (\overrightarrow{\nabla}\varphi(x))^2 + m^2\varphi(x)^2] + g\,\varphi(x)^4\}\,d^3x. \tag{2.2}$$

It is important that Monte Carlo studies (by setting $\hbar = 1$) have confirmed the correct results of this solution because it had the $\hbar$-term in its Hamiltonian. While canonical quantization has failed to solve this problem, affine quantization has now completely, and correctly, solved $\varphi(x)^4$; see [1–3].

Conclusions

For more than a century, we have had canonical quantization as the tool of quantization itself in which we had only Q and P along with $QP - PQ = i\hbar\mathbb{1}$ and related similar field operators which require space range from plus and minus infinity. This has forced such quantization requiring these infinite properties, but on forcefully some things need non-infinite properties to solve certain formulations. A common example of what cannot deal with canonical quantization is the "particle in a box", which is properly reached using some new proper quantization, that is now available under the name of affine quantization [4]. This new quantization tool can be used to solve new kinds of properties of length in the point of view in which the new procedures can use the "particle in the box" as a first example of a well developed approach to quantization. This new procedure called affine quantization can tackle a marvelous combination of problems. This paper has been chosen to tackle some examples that cannot be solved by canonical quantization, but instead requires the proper procedures leading by affine quantization. This new tool can solve the correct quantization of a grand supply of soluble problems. See for yourselves!

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